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# The Statistical Distribution of the Number of Prompt Fission Gamma Rays

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Peelle and Maienschein, among others, have reported that on average there are seven prompt gamma rays per fission of $^{235}$U. If we assume that the number of prompt gamma rays per fission is a random variable then we can consider the number of eight gamma rays as the first term of a moment expansion of the appropriate distribution. Higher order moments of this distribution are related to the number of n-fold coincidences of fission gamma rays as determined with appropriate gamma detectors. Measurements of twofold, threefold and fourfold coincidences are considered. The statistical theory of these coincidences is developed in this paper. Experimental work is underway, but final results have not yet been obtained.

***KEYWORDS: fission, prompt fission gamma rays.***

## I. Introduction

The experimental verification of the Madland-Nix[1] theory of nuclear fission requires *inter alia* the determination of neutron energy spectra following fission induced by primary neutrons in the thermal-to-3MeV energy range. Staples[2] and coworkers have measured these spectra for fission induced in $^{235}$U by low energy neutrons so that they could use energy discrimination to distinguish fission neutrons from primary neutrons. Staples' approach can be extended to higher primary neutron energies, provided a way is found to distinguish fission neutrons from scattered primary neutrons. Woodring *et al.*[3] have used the occurrence of prompt fission gamma rays as an indication that a fission event has occurred. They distinguish fission gamma rays from those due to inelastic neutron scattering by their multiplicity. Early work by Peele and Maienschein[4] indicates an average prompt fission gamma ray energy per fission close to one MeV with an average number of gamma rays per fission of about seven. (For additional publications see Wagemans[5]) This number is far larger than what one would expect from neutron inelastic scattering. Thus Woodring uses three gamma detectors to detect coincident gamma rays in his experiment and assumes a coincident neutron to be a fission neutron. This assumption is in need of experimental verification and that is an aim of this paper. In addition the paper suggests a way to obtain the second moment of the gamma ray distribution

## II. Theory

Consider a subnanosecond pulsed proton beam incident upon a neutron producing target and assume further that the neutrons irradiate a fissile sample, e. g. $^{235}$U. This sample is surrounded by four $BaF_2$ gamma detectors, in close proximity to the fissile sample, and which are shielded against neutrons and gamma rays from the target. (The number of detectors, four, is not critical and may be changed if desired.) Denote by $A_n$ the probability that in one fission event $n$ prompt gamma quanta are emitted and construct the gamma generating function[6], $f(\varepsilon)$

$$f(\varepsilon)=\sum_{n=0}^{\infty}A_n\varepsilon^n \tag{1}$$

Note that $f$ is normalized, $f(1)=1$. The gamma detectors have a detection efficiency $\eta$, assumed to be the same for the four devices and each detector subtends the same solid angle $\Delta\Omega$. We combine these parameters to form

$$a=\eta\Delta\Omega/4\pi \tag{2}$$

For each binary efficiency generating function $\gamma_i$ can be introduced,

$$\gamma_i=1-a+a\varepsilon_i, \quad i=1,2,3,4, \tag{3}$$

which is normalized to 1. Since the gamma detection probability of any one detector is independent of that of any other detector we can construct an overall gamma detection generating function, for one gamma quantum,

$$g(\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)=(1-4a)+a(\varepsilon_1+\varepsilon_2+\varepsilon_3+\varepsilon_4). \tag{4}$$

If we combine $f$ and $g$ we obtain the overall generating function $G$, where

$$G=f\{g(\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4)\}=G(\varepsilon_1,\ldots\varepsilon_4)\,. \tag{5}$$

When expanded in a power series $G$ can be written as

$$G=\sum_{l,m,n,o=0}^{\infty}b_{l,m,n,o}\varepsilon_1^{\,l}\varepsilon_2^{\,m}\varepsilon_3^{\,n}\varepsilon_4^{\,o} \tag{6}$$

here $b_{l,m,n,o}$ is the probability that detector 1 detects $l$ gamma rays, 2 $m$, 3 $n$, and detector 4 $o$ gamma rays. We do not need

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to perform the expansion; we note the following convenient results: $G(0,0,0,0)$ is the probability that none of the four detectors records a gamma ray. For brevity we write

$$G(0,0,0,0) = b_{0,0,0,0} = b_0 \tag{7}$$

The probability that e.g. detector 1 detects (at least) one gamma ray while the others detect none is designated by $S_1$, the singles probability of detector 1,

$$S_1 = G(1,0,0,0) - b_0 = \sum_{l=1}^{\infty} b_{l,0,0,0} \tag{8}$$

$S_2$, $S_3$, and $S_4$ are defined in analogous form ; they are equal to $S_1$. The total singles probability $S$ is

$$S=S_1+S_2+S_3+S_4=4S_1. \tag{9}$$

The doubles probability, $D$, is defined in a similar way. We designate by $D_{12}$ the probability that detectors 1 and 2 record at least one gamma ray each while 3 and 4 record none. Explicitly,

$$D_{12} = \sum_{l,m=1}^{\infty} b_{l,m,0,0} \tag{10}$$

$D_{12}$ resembles $G(1,1,0,0)$, note that this $G$ contains $S$-terms too. Subtraction of these terms leads to

$$D_{12} = G(1,1,0,0) - S_1 - S_2 - b_0 \tag{11}$$

There are six doubles terms so that the overall doubles probability, $D$, is

$$D = D_{12} + D_{13} + D_{14} + D_{23} + D_{24} + D_{34} \tag{12}$$

or

$$D = 6D_{12} \tag{13}$$

The triples and quadruples probabilities, $T$ and $Q$, are found in a similar fashion. Starting from

$$G(1,1,1,0) = b_0 + 3S_1 + 3D_{12} + T_{123} \tag{14}$$

we obtain

$$T = 4T_{123} = 4G(1,1,1,0) - 4b_0 - 3S - 2D \tag{15}$$

and, analogously, it follows from

$$G(1,1,1,1) = 1 = b_0 + S + D + T + Q \tag{16}$$

that

$$Q = 1 - b_0 - S - D - T \tag{17}$$

## III. Application

In the following we designate by $\lambda$ the average number of prompt gamma rays emitted following fission. We shall consider two different distributions for $f(\varepsilon)$: A Poisson distribution and a sharp distribution. For the Poisson distribution we have

$$A_n = e^{-\lambda}\frac{\lambda^n}{n!} \tag{18}$$

$$f(\varepsilon) = e^{\lambda(\varepsilon-1)} \tag{19}$$

$$G = \exp\{\lambda a(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4 - 4)\} \tag{20}$$

In this case $b_0$, $S$, $D$, $T$, and $Q$ are given by the following expressions:

$$b_0 = \exp\{-4\lambda a\} \tag{21}$$

$$S = 4(\exp\{-3\lambda a\} - \exp\{-4\lambda a\}) \tag{22}$$

$$D = 6(\exp\{-2\lambda a\} - 2\exp\{-3\lambda a\} + \exp\{-4\lambda a\}) \tag{23}$$

$$T = 4(\exp\{-\lambda a\} - 3\exp\{-2\lambda a\} + 3\exp\{-3\lambda a\} - \exp\{-4\lambda a\}) \tag{24}$$

and

$$Q = 1 - 4\exp\{-\lambda a\} + 6\exp\{-2\lambda a\} - 4\exp\{-3\lambda a\} + \exp\{-4\lambda a\}) \tag{25}$$

On the other hand a sharp distribution implies that $A_\lambda \neq 0$ while all other $A_n = 0$. In this case

$$f(\varepsilon) = \varepsilon^{\lambda} \tag{26}$$

$$G = (\{1-4a\} + a\{\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \varepsilon_4\})^{\lambda} \tag{27}$$

$$b_0 = \{1-4a\}^{\lambda} \tag{28}$$

$$S = 4(\{1-3a\}^{\lambda} - \{1-4a\}^{\lambda}) \tag{29}$$

$$D = 6(\{1-2a\}^{\lambda} - 2\{1-3a\}^{\lambda} + \{1-4a\}^{\lambda}) \quad (30)$$

$$T = 4(\{1-a\}^{\lambda} - 3\{1-2a\}^{\lambda} + 3\{1-3a\}^{\lambda} - \{1-4a\}^{\lambda}) \quad (31)$$

$$Q = 1 - 4\{1-a\}^{\lambda} + 6\{1-2a\}^{\lambda} - 4\{1-3a\}^{\lambda} + \{1-4a\}^{\lambda} \quad (32)$$

## IV. Discussion

It is interesting to compare these results with numerical data one might expect to obtain in an experiment. Consider four cylindrical scintillation detectors (radii $R$) located co-axially on the positive and negative $x$- and $y$-axes of a Carthesian coordinate system, so that their front surfaces are at a distance $R$ from the origin. The fissile sample is at the origin. Neutrons are incident from the $+z$ direction, reach the sample, and proceed to the dump at in the $-z$ direction. With this geometry the solid angle subtended by the detectors, $\Delta\Omega$, is

$$\Delta\Omega = 2\pi(1 - 1/\sqrt{2}) \quad (33)$$

We assume that the detector efficiencies are $\eta = 1$ and we take $\lambda = 7$, so that $a = 0.1416$. Now consider three different distributions, the sharp distribution, the Poisson distribution, both discussed in the previous section, and a broad distribution with $A_n = 1/15$, for $0 \le n \le 14$, and $A_n = 0$ for $n > 14$. All three distributions have the same average value of seven. In **Table 1** we list the parameters $b_0$, $S$, $D$, $T$, $Q$, $Q/T$ for the three distributions together with the mean square deviation, $MSD$, for each distribution. The correlation between $Q/T$ and $MSD$ is evident. The values for $Q/T$ in this table may be compared with experimental data to decide if the $MSD$ of the number-of-gamma-rays distribution is larger, equal to, or smaller than that of a Poisson distribution.

**Table 1.** Comparison of the three distributions.

| | Sharp Distribution | Poisson Distribution | Broad Distribution |
|---|---|---|---|
| $b_0$ | $2.099\times10^{-3}$ | $1.657\times10^{-2}$ | 0.1138 |
| $S$ | $6.128\times10^{-2}$ | 0.1185 | 0.1517 |
| $D$ | 0.3341 | 0.3176 | 0.2205 |
| $T$ | 0.4601 | 0.3783 | 0.3006 |
| $Q$ | 0.1423 | 0.1690 | 0.2134 |
| $Q/T$ | 0.3093 | 0.4468 | 0.7101 |
| $MSD$ | 0 | 7 | 18.67 |


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