Journal of NUCLEAR SCIENCE and TECHNOLOGY, Supplement 2, p. 726–729 (August 2002)

Cross Sections for Multistep Direct Reactions

Paraskevi DEMETRIOU[1,*], Andrzej MARCINKOWSKI[2], Bohdan MARIAŃSKI[2]

[1]*Institute of Nuclear Physics, NCSR "Demokritos", GR-15310 Athens, Greece*

[2]*The Andrzej Soltan Institute for Nuclear Studies, 00-681 Warsaw, Hoża 69, Poland*

Inelastic scattering and charge-exchange reactions have been analysed at energies ranging from 14 to 27 MeV using the modified multistep direct reaction theory (MSD) of Feshbach, Kerman and Koonin. [1)] The modified theory considers the non-DWBA matrix elements in the MSD cross section formulae and includes both incoherent particle-hole excitations and coherent collective excitations in the continuum, according to the prescription of. [2,3)] The results show important contributions from multistep processes at all energies considered.

KEYWORDS: nucleon inelastic scattering ^{93}Nb(n,n'), E=14-26 MeV, nuclear reactions ^{65}Cu(p,n), E=27 MeV, ^{100}Mo(p,n), E=26 MeV; multistep reaction cross sections

I. Introduction

In recent years quantum mechanical theories have been developed to describe the multistep direct (MSD) and multistep compound (MSC) processes[4–6)] and have proved to be able to account for both experimental energy spectra and angular distributions with considerable accuracy. The statistical multistep direct (MSD) theory of Feshbach, Kerman and Koonin (FKK),[1)] in particular, has been used in many analyses of nucleon-induced reactions due to the appealing convolution structure of the multistep cross sections that makes the computation of higher order processes practicable. However, the FKK theory has for quite some time been a subject of controversy due to the biorthogonality of the distorted waves involved in the transition amplitudes describing the multistep cross sections.[7)] The biorthogonally conjugated wave functions lead to non-DWBA transition matrix elements, that can be expressed in terms of normal DWBA matrix elements multiplied by the inverse elastic scattering S-matrix.[8)] It has been argued that these enhancing inverse S-factors are washed out by averaging over energy in the continuum.[9,10)] As a result normal DWBA matrix elements are still commonly used in practical calculations. Almost all of the analyses of inelastic scattering and charge-exchange reactions using normal DWBA matrix elements have concluded that nucleon emission at low energies can be described as a one-step direct reaction mainly. On the other hand, it has been shown that the limits imposed by the energy weighted sum rules (EWSR's) on transitions of given angular momentum transfer lead to a significant reduction of the one-step cross section.[2)] In order to compensate for the reduced one-step cross sections, the enhanced MSD cross sections obtained with the use of non-DWBA matrix elements are required.

In this paper we present the results of the FKK theory using non-DWBA matrix elements for neutron inelastic scattering on niobium at incident energies of 14-26 MeV, and for the charge-exchange reaction (p,n) on copper and molybdenum at incident energies of about 26 MeV. The theory is modified to include the excitation of collective vibrations, in addition to the excitation of incoherent particle-hole pairs, according to the prescription of.[2,3)] When all contributions are taken into account, the resulting cross sections are able to reproduce the data over the whole energy range and for both types of reaction considered.

* Corresponding author, Tel. +30-1-6503514, Fax. +30- 1-65011215, E-mail: vivian@mail.demokritos.gr

II. Formalism

The cross section of the MSD reaction is given by a convolution of 1SD cross sections,[1)]

$$\left(\frac{d^2\sigma}{dEd\Omega}\right)_{MSD} = \int \frac{m_1E_1}{(2\pi)^2\hbar^2}dE_1d\Omega_1 \int \frac{m_2E_2}{(2\pi)^2\hbar^2}dE_2d\Omega_2 \ldots \times \int \frac{m_nE_n}{(2\pi)^2\hbar^2}dE_nd\Omega_n \left(\frac{d^2\sigma}{dE_nd\Omega_n}\right)^{1SD} \ldots \times \left(\frac{d^2\sigma}{dE_2d\Omega_2}\right)^{1SD} \left(\frac{d^2\sigma}{dE_1d\Omega_1}\right)^{1SD} \quad (1)$$

where E_n, Ω_n and m_n are the energy, solid angle and mass of the scattered particle after the n-th stage of the reaction. The matrix elements describing the successive 1SD transitions Eq. (1) involve, except for the last one, the biorthogonally conjugated distorted waves. The corresponding non-DWBA matrix elements can be expressed in terms of the normal DWBA matrix elements including an inverse elastic scattering S-matrix factor. Therefore the corresponding 1SD cross sections in Eq. (1) are given by the partial ℓ-wave cross sections multiplied by the modulus squared of the inverse elastic matrix elements S_ℓ^{-2}. The elastic scattering matrix elements are related to the partial wave transmission coefficients T_ℓ of the optical potential by $|S_\ell|^2 = 1 - T_\ell$. The S_ℓ^{-2} factors may become divergent at the positions of the single-particle resonances[8)] leading thus to an unphysical ehnancement in the non-DWBA cross sections. These anomalies are removed by assuming an average behaviour of the corresponding S_ℓ^{-2}'s.

The 1SD cross sections entering Eq. (1) were derived by imposing the restrictions of the EWSR's on transitions of different multipolarity or angular momentum transfer. These restrictions split the 1SD cross sections into two terms, (i) due to the collective vibrations of multipolarity $\lambda \leq 4$ and (ii) due to the incoherent excitation of the *ph*-pairs of angular momen-

tum transfer ℓ <4.[2] The 1SD cross section is thus given by

$$\left(\frac{d^2\sigma}{dUd\Omega}\right)_{1SD} = \sum_{n(\tau,\lambda\leq 4)} \epsilon_{\alpha,\beta}\beta^2_{\lambda,\tau}(n)\left(\frac{J_{0,\tau}}{J_{0,0}}\right)^2\left(\frac{d\sigma}{d\Omega}\right)^{macr}_{\lambda} \times f[\hbar\omega_\lambda(n),\Gamma] + \sum_{\alpha,\beta}\sum_{\ell>4}(2\ell+1)Pg_\pi^{p_\pi+h_\pi}g_\nu^{p_\nu+h_\nu}U \times R_{1,1}(\ell)V^2_{\alpha\beta}\left\langle\left(\frac{d\sigma}{d\Omega}\right)^{micr}_{\ell}\right\rangle .$$

Only the dominating transitions with spin transfer s=0 are allowed for in Eq. (2). The macroscopic DWBA cross sections describe the excited collective one-phonon states n with energy $\hbar\omega_\lambda$, multipolarity λ, deformation parameter $\beta_{\lambda,\tau}$ and isospin transfer τ. The $\epsilon_{\alpha,\beta}$ term is $\epsilon_{\pi,\pi} = \epsilon_{\nu,\nu} = 1$ for inelastic scattering and $\epsilon_{\pi,\nu} = \epsilon_{\nu,\pi} = 2$ for charge-exchange reactions. The ratio of terms in the nucleon-nucleon effective interaction ($J_{0,1}/J_{0,0}$) is close to -1/2.[11] The term f_λ is the energy distribution function which is assumed to be Gaussian with a width Γ reflecting the experimental energy resolution for the low-energy collective levels, or a Lorentzian with width typical of the giant resonances in the continuum. The effective β_λ parameters for giant resonances can be obtained by depleting the energy-weighted sum rules (EWSR's) by the strengths of all known individual collective levels of a given multipolarity λ. The sum over α and β in the second term of Eq. (2) contains two terms for nucleon scattering that correspond to the excitation of a neutron ph-pair and a proton ph-pair. The sum reduces to a single term for charge-exchange reactions. The macroscopic and microscopic DWBA cross sections were calculated with the DWUCK-4 code[12] using a macroscopic form factor and a two-particle form factor, respectively. In the latter case, a real effective interaction of Yukawa form and strengths $V_{\pi,\pi} = V_{\nu,\nu} = 12.7$ MeV and $V_{\pi,\nu} = V_{\nu,\pi} = 43.1$ MeV. from[13] was used. The parameters involved in the calculations of the 1SD cross sections can be found in more detail in.[3,14-16]

Since the enhanced 1SD cross section of Eq. (2) consist of the collective term (S^{-2}vib) and the incoherent term (S^{-2}ph), the multistep cross sections of Eq. (1) will contain the following combinations:
1SD, (vib)+(ph),
2SD, (S^{-2}vib,vib)+(S^{-2}ph,vib)+(S^{-2}vib,ph)+(S^{-2}ph,ph)
3SD, (S^{-2}vib,S^{-2}vib,vib)+(S^{-2}ph,S^{-2}vib,vib)+
(S^{-2}vib,S^{-2}ph,vib)+(S^{-2}vib,S^{-2}vib,ph)+(S^{-2}ph,S^{-2}ph,vib)+
(S^{-2}ph,S^{-2}vib,ph)+(S^{-2}vib,S^{-2}ph,ph)+(S^{-2}ph,S^{-2}ph,ph)
etc.

In the calculations of the MSD cross sections for inelastic scattering of neutrons, we have considered contributions from process where the leading particle in the intermediate stages is a neutron: (nnS^{-2},nn), (nnS^{-2},nnS^{-2},nn) and (nnS^{-2},nnS^{-2},nnS^{-2},nn), where (nnS^{-2}) = $\sum_\ell \sigma_\ell$(nn)S^{-2}_ℓ. We have ignored terms containing proton scattering and charge-exchange of the leading particle since they are significantly smaller than those of neutron scattering. However, this may not be true for charge-exchange reactions so in the latter case we calculate all the contributing sequences of collisions:[16]
1SD, (pn)
2SD, (pnS^{-2},nn)+(ppS^{-2},pn)
3SD, (pnS^{-2},nnS^{-2},nn)+(ppS^{-2},pnS^{-2},nn)+
(ppS^{-2},ppS^{-2},pn)+(pnS^{-2},npS^{-2},pn),
etc.

III. Results and Discussion

(2) **Inelastic Neutron Scattering**: In **Table 1** we show the summed 1SD, 2SD, 3SD and 4SD cross sections integrated over outgoing energy and angle at the same incident energies. These cross sections are compared with the inclusive neutron emission spectra of ^{93}Nb(n,n') in **Figs. 1, 2, 3** for the incident energies of 14, 20 and 26 MeV, respectively. The broad structure present in the 1SD spectrum at high outgoing energies is due to the low-energy one-phonon states. This one-phonon structure is then folded in Eq. (1) into multiphonon states and appears in the 2SD and 3SD spectra at excitation energies twice and trice larger than the energy of the one-phonon states, respectively. The calculations of the compound nucleus (CN) and statistical multistep compound (MSC) emission are described in.[3] A nice agreement is found between theory and experiment at all incident energies. The results of the calculations reveal substantial contributions from multistep processes even at the low incident energy of 14 MeV.

Table 1 The 1SD and MSD cross sections integrated over energy and angle for the ^{93}Nb(n,n'')^{93}Nb reaction at energies 14, 20 and 26 MeV.

	14 MeV	20 MeV	26 MeV
σ(1SD) (mb)	205	293	354
σ(2SD) (mb)	54	187	215
σ(3SD) (mb)	12.3	70	96
σ(4SD) (mb)	2	20	36

Charge-exchange Reactions: The results of the calculations reveal substantial contributions from multistep processes. The 2SD, 3SD and 4SD integral cross sections for ^{65}Cu are 38 mb, 13.6 mb and 4.3 mb, respectively, compared to the 1SD = (vib)+(ph) cross section that amounts to 74 mb. For ^{100}Mo the corresponding results for 2SD, 3SD and 4SD cross sections are 55mb, 34.8 mb and 20 mb, respectively, compared to the 1SD = (vib)+(ph) = 74 mb. The cross sections of the sequences of the leading particle collisions that contribute to the 2SD, 3SD and 4SD cross sections are presented in **Tables 2** for the ^{100}Mo(p,n) reactions. The largest contributions are due to the occurence of neutron inelastic scattering in the sequence of collisions. However, no single sequence is sufficient to describe the multistep cross section. The summed 1SD, 2SD, 3SD and 4SD cross sections give a good description of the inclusive emission spectrum of the ^{65}Cu(p,n) and ^{100}Mo(p,n) reactions as can be seen in **Figs. 4, 5**, respectively.

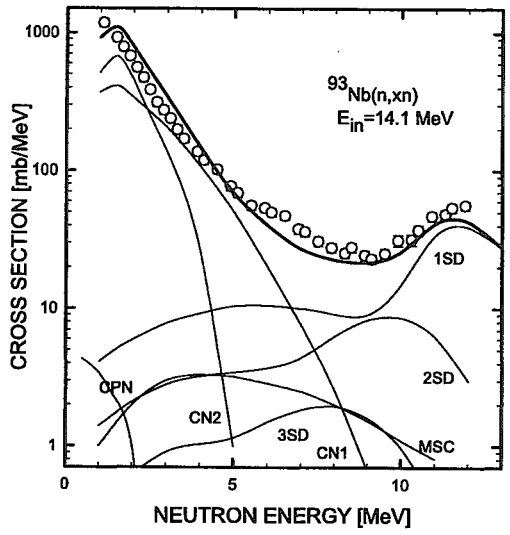

Fig. 1 Comparison of the calculated cross sections with the spectrum of neutrons from the ^{93}Nb(n,n')^{93}Nb reaction measured at incident energy of 14 MeV.[17] The thick line is a sum of all contributions shown. The labels CN1, CN2 and CN3 denote the primary, secondary and tertiary neutrons evaporated from the compound nucleus, respectively. CPN denotes secondary neutrons preceded by evaporation of a proton and MSC labels the sum of emissions from the three steps of the preequilibrium compound reaction.

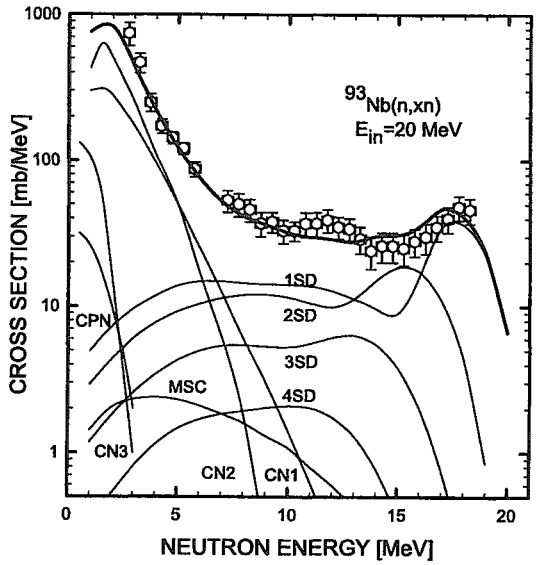

Fig. 2 Same as in **Fig. 1** but for 20 MeV.[18]

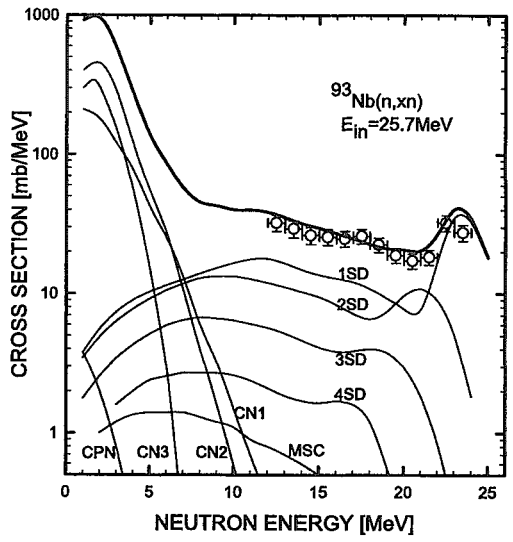

Fig. 3 Same as in **Fig. 1** but for 25.7.[19]

Table 2 The sequences of the leading particle reaction stages contributing to the 2SD, 3SD and 4SD ^{100}Mo(p,n)^{100}Tc reactions at 26 MeV.

σ(2SD)	(mb)	σ(3SD)	(mb)
(pnS^{-2},nn)	46	(pnS^{-2},nnS^{-2},nn)	26.5
(ppS^{-2},pn)	9	(ppS^{-2},pnS^{-2},nn)	4.9
		(ppS^{-2},ppS^{-2},pn)	3.3
		(pnS^{-2},npS^{-2},pn)	0.07
Total	55		34.8

σ(4SD)	(mb)
(pnS^{-2},nnS^{-2},nnS^{-2},nn)	14.5
(ppS^{-2},pnS^{-2},nnS^{-2},nn)	2.5
(ppS^{-2},ppS^{-2},pnS^{-2},nn)	1.8
(pnS^{-2},npS^{-2},pnS^{-2},nn)	0.75
(pnS^{-2},nnS^{-2},npS^{-2},pn)	0.03
(ppS^{-2},ppS^{-2},ppS^{-2},pn)	0.03
(ppS^{-2},pnS^{-2},npS^{-2},pn)	0.01
(pnS^{-2},npS^{-2},ppS^{-2},pn)	0.004
Total	20

IV. Conclusion

The multistep direct theory of FKK[1] has been modified to include non-DWBA matrix elements in the description of the multistep transitions.[7] Combined with the new 1SD cross sections obtained by imposing restrictions from EWSR's[2] it is able to reproduce neutron inelastic scattering spectra at energies 14-26 MeV, as well as charge-exchange reaction spectra at energy about 26 MeV. An overall enhancement of the 2SD, 3SD and 4SD cross sections is found for both types of reactions. For neutron inelastic scattering the enhancement of the MSD cross sections is of the order of $(3.5)^{(M-1)}$. In the case of the charge-exchange (p,n) reaction it is found that the main contributions come from multistep processes involving neutron scattering in the sequence of reaction steps.

Acknowledgment

This work was performed under the Greek-Polish bilateral agreement for scientific cooperation No-028/2001-2002, 3326/R01/R02. P.D. acknowledges partial support of the Greek Atomic Energy Agencyfor attending the conference.

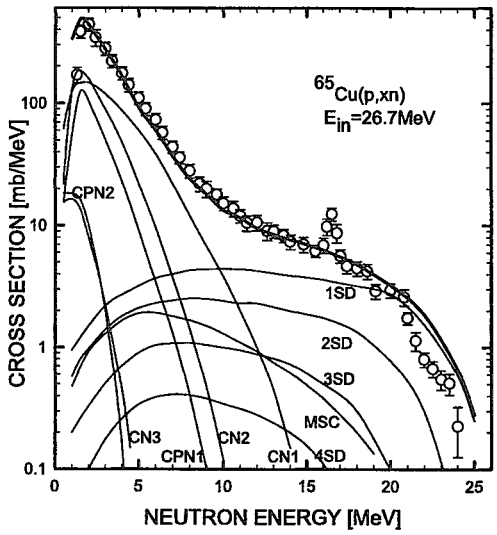

Fig. 4 Comparison with the spectrum of neutrons from the ^{65}Cu(p,n)^{65}Zn reaction measured at incident energy of 26.7 MeV.[20] The thick line is the sum of all contributions shown. The labels CN1, CN2, CN3 denote the primary, secondary and tertiary neutrons evaporated from the compound nucleus, respectively. CPN1 and CPN2 denotes secondary and tertiary neutrons preceded by a proton and MSC labels the sum of contributions from the three steps of preequilibrium compound reaction.

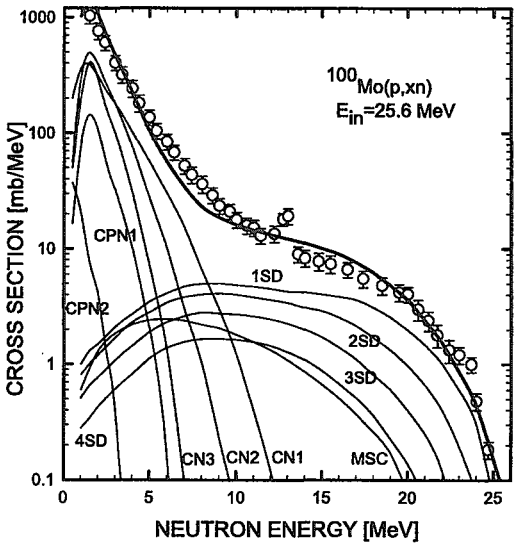

Fig. 5 Same as in **Fig. 4** but for the ^{100}Mo(p,n)^{100}Tc reaction at incident energy of 25.6 MeV.[21]

References

1) H. Feshbach, A. Kerman, S. Koonin, "The Statistical Theory of Multi-Step Compound and Direct Reactions", *Ann. Phys. (N.Y.)*, **125**, 429 (1980).
2) A. Marcinkowski, B. Mariański, "Analysis of low-energy direct reactions and the novel one-step cross section", *Nucl. Phys.*, **A653**, 3 (1999).
3) P. Demetriou, A. Marcinkowski, B. Mariański, "Multistep Direct Reactions at Low Energies", *Phys. Lett.*, **B493**, 281 (2000).
4) E. Gadioli, P.E. Hodgson, *Pre-equilibrium Reactions* (Clarendon, Oxford, 1992).
5) T. Tamura, T. Udagawa, H. Lenske, "Multistep direct reaction analysis of continuum spectra in reactions induced by light ions", *Phys. Rev.*, **C26**, 379 (1982).
6) H. Nishioka, H.A. Weidenmüller, S. Yoshida, "Statistical Theory of Precompound Reactions: The Multistep Direct Process", *Ann. Phys.*, **183**, 166 (1988); H. Nishioka, J.J.M. Verbaarschot, H.A. Weidenmüller, S. Yoshida, *Ann. Phys.*, **172**, 67 (1986).
7) M.S. Hussein, R. Bonetti, "The Statistical Multistep Direct Emission Theory Revised", *Phys. Lett.*, **B112**, 189 (1982).
8) I. Kumabe, M. Haruta, M. Hyakutake, M. Matoba, "COMMENT ON APPLICATION OF STATISTICAL MULTI-STEP DIRECT EMISSION THEORY TO A NEUTRON-EMITTING REACTION", *Phys. Lett.*, **B140**, 272 (1984).
9) H. Feshbach, "Energy Averaging and the Statistical Multistep Direct Process", *Ann. Phys. (N.Y.)*, **159**, 150 (1985).
10) H. Feshbach, "Statistical multistep direct reaction," *Phys. Rev.*, **C48**, R2253 (1994).
11) G.A. Needham, F.P. Brady, D.H. Fitzgerald, J.L. Romero, J.L. Ullman, J.W. Watson, C. Zanelli, N.S.P. King, G.R. Satchler, "EXCITATION OF ANALOG ISOVECTOR GIANT RESONANCES VIA THE (p,n) REACTION ON ^{14}N AND ^{16}O AT 60 MeV", *Nucl. Phys.*, **A385**, 349 (1982).
12) P.D. Kunz, E. Rost, *Computational Nuclear Physics*, **2**, ed. K. Langanke et al. (Springer, Berlin, 1993) Ch. 5
13) S.M. Austin, in *Proc. Conf. on (p, n) Reactions and the Nucleon-Nucleon Force*, Telluride, Colorado, 1979, Eds. C.D. Goodman et al.
14) A. Marcinkowski, P. Demetriou, B. Mariański, "Cross sections for multistep direct neutron scattering on niobium", *Nucl. Phys.*, **A694**, 312 (2001).
15) P. Demetriou, A. Marcinkowski, B. Mariański, "Multistep Direct Reactions of 14 MeV Neutrons", *Acta Phys. Pol.*, **B32**, 3003 (2001).
16) P. Demetriou, A. Marcinkowski, B. Mariański, "Multistep processes in charge-exchange reactions", *Nucl. Phys.*, **A697**, 171 (2002).
17) A. Takahashi, M. Gotoh, Y. Sasaki, H. Sugimoto, "Double and Single Differential Neutron Emission Cross Sections at 14.1 MeV", *OKTAVIAN Report* bf A-92-01, Vol.2, Osaka University.
18) A. Marcinkowski, J. Rapaport, R.W. Finlay, X. Aslanoglou, D. Kielan, "Neutron Emission Cross Section on ^{93}Nb and ^{209}Bi at 20 MeV Incident Energy", *Nucl. Phys.*, **A530**, 75 (1991).
19) A. Marcinkowski, R.W. Finlay, G. Randers-Pehrson, C.E. Brient, R. Kurup, S. Mellena, A. Meigooni, R. Tailor, "Neutron Emission Cross Sections at 25.7 MeV: ^{51}V, ^{56}Fe, ^{65}Cu, ^{93}Nb, and ^{209}Bi", *Nucl. Sci. Eng.*, **83**, 13 (1983).
20) Y. Holler, A. Kaminsky, R. Langkau, W. Scobel, M. Trabandt, "Pre-Compound Emission in the Reaction ^{65}Cu(p,xn) and the Nucleon-Nucleon Scattering Mechanism", *Nucl. Phys.*, **A442**, 79 (1985).
21) E. Mordhorst, M. Trabandt, A. Kaminsky, H. Krause, W. Scobel, R. Bonetti, F. Crespi, "Pairing effects and multistep direct and compound emission in the $^{92-100}$Mo(p,xn) reaction", *Phys. Rev.*, **C34**, 103 (1986).